Solutions HCT 2A

Question 1

\[ \tan \alpha = \left( \frac{1.718 - 0.526}{1} \right) = 1.1920 \]
\[ \therefore \alpha = 50.0^\circ \]

**FBD of B**

**Equilibrium equations**

\[ \sum F_x = 0 \implies T_{AB} \cos 50^\circ = T_{BC} \cos 40^\circ \]  
\[ \sum F_x = 0 \implies T_{AB} \sin 50^\circ + T_{BC} \sin 40^\circ = W \]

**Solve system of equations**

From (1): \( T_{AB} = 1.192T_{BC} \)  
Substitute in (2): \( 1.192T_{BC} \sin 50^\circ + T_{BC} \sin 40^\circ = W \)
\[ \therefore 1.556T_{BC} = W \]

**Answer to the question**

\( T_{AB} \leq 92.2 \, N \) and \( T_{BC} \leq 92.2 \, N \) must hold, and \( T_{AB} = 1.192T_{BC} > T_{BC} \). Therefore we choose \( T_{AB} = 92.2 \, N \) and solve for \( T_{BC} \) and \( W \):

From (3), \( T_{BC} = T_{AB}/1.192 = 77.36 \, N \). Substitute into (4) to find \( W = 1.556T_{BC} = 120.4 \, N \).

Hence, the maximum mass is \( m = 120.4 \, N/9.8 \, m/s^2 = 12.3 \, kg \).

**Question 2**

The moment of \( \mathbf{F} \) about point \( B \) is \( \overline{M}_B = \overline{BD} \times \mathbf{F} \).

\[
\overline{M}_B = \begin{vmatrix}
7 & 7 & 0 \\
56 & 65 & 20 \\
-10 & 15 & 0
\end{vmatrix}
Ncm
= \left( \left\{(56)(15) - (65)(-10)\right\}\hat{i} - \left\{(0)(15) - (65)(20)\right\}\hat{j} + \left\{(0)(-10) - (56)(20)\right\}\hat{k} \right) Ncm
= (14.9\hat{i} + 13\hat{j} - 11.2\hat{k}) \, Nm
\]

The moment of \( \mathbf{F} \) about point \( D \) is \( \overline{M}_D = \overline{0} \), since \( D \) lies on the line of action of \( \mathbf{F} \).
Solutions HCT 2B

Question 1

\[ \tan \alpha = \frac{1.365 - 0.526}{1} = 0.8390 \]
\[ \therefore \alpha = 40.0^\circ \]

FBD of \( B \)

Equilibrium equations

\[ \sum F_x = 0 \implies T_{AB} \cos 40^\circ = T_{BC} \cos 50^\circ \] (1)
\[ \sum F_x = 0 \implies T_{AB} \sin 40^\circ + T_{BC} \sin 50^\circ = W \] (2)

Solve system of equations

From (1): \( T_{AB} = 0.8391T_{BC} \) (3)
Substitute in (2): \( 0.8391T_{BC} \sin 40^\circ + T_{BC} \sin 50^\circ = W \)
\[ \therefore 1.305T_{BC} = W \] (4)

Answer to the question

\( T_{AB} \leq 78.6 \, N \) and \( T_{BC} \leq 78.6 \, N \) must hold, and \( T_{AB} = 0.8391T_{BC} < T_{BC} \). Therefore we choose \( T_{BC} = 78.6 \, N \) and solve for \( W \) from (4):
Substitute into (4) to find \( W = 1.305T_{BC} = 102.6 \, N \).

Hence, the maximum mass is \( m = 102.6 \, N/9.8 \, m/s^2 = 10.5 \, kg \).

Question 2

The moment of \( \bar{F} \) about point \( A \) is \( \overrightarrow{M}_A = \overrightarrow{AC} \times \bar{F} \).

\[
\overrightarrow{M}_B = \begin{vmatrix}
7 & 7 & k \\
0 & 65 & 56 \\
10 & -15 & 20
\end{vmatrix}
\]
\[ = \left( \{(65)(20) - (56)(-15)\}7 - \{(0)(20) - (56)(10)\}7 + \{(0)(-15) - (65)(10)\}1 \right) \text{Ncm} \]
\[ = \left( 21.47 + 5.607 - 6.50k \right) \text{Nm} \]

The moment of \( \bar{F} \) about point \( C \) is \( \overrightarrow{M}_C = \bar{0} \), since \( C \) lies on the line of action of \( \bar{F} \).
Solutions HCT 2C

Question 1

FBD of $B$

Equilibrium equations

\[ \sum F_x = 0 \implies T_{AB} \cos 40^\circ = T_{BC} \cos 40^\circ \quad (1) \]
\[ \sum F_x = 0 \implies T_{AB} \sin 40^\circ + T_{BC} \sin 40^\circ = W \quad (2) \]

Solve system of equations

From (1): $T_{AB} = T_{BC}$ \quad (3)
Substitute in (2): $2T_{BC} \sin 40^\circ = W$
\[ \therefore 1.286T_{BC} = W \quad (4) \]

Answer to the question

$T_{AB} \leq 63.8\, N$ and $T_{BC} \leq 63.8\, N$ must hold, and $T_{AB} = T_{BC}$ (from (3)). Therefore cables $AB$ and $BC$ attain the maximum tension simultaneously and we choose $T_{AB} = T_{BC} = 63.8\, N$.

Substitute into (4) to find $W = 1.286T_{BC} = 82.02\, N$.

Hence, the maximum mass is $m = \frac{82.02\, N}{9.8\, m/s^2} = 8.37\, kg$.

Question 2

The moment of $\mathbf{F}$ about point $A$ is $\mathbf{M}_A = \overrightarrow{AC} \times \mathbf{F}$.

\[ \mathbf{M}_B = \begin{vmatrix} i & j & k \\ 7 & 28 & 53 \\ -20 & 10 & 15 \end{vmatrix} \, Ncm \]
\[ = \left(\{(28)(15) - (53)(10)\}i - \{(0)(15) - (53)(-20)\}j + \{(0)(10) - (28)(-20)\}k\right) \, Ncm \]
\[ = (-1.10i - 10.6j + 5.60k) \, Nm \]

The moment of $\mathbf{F}$ about point $C$ is $\mathbf{M}_C = \overrightarrow{0}$, since $C$ lies on the line of action of $\mathbf{F}$. 

\[
\tan \alpha = \left(\frac{1.365 - 0.526}{1}\right) = 0.8390
\]
\[ \therefore \alpha = 40.0^\circ \]