SWK 122 Semester 2 2012

Solutions for semester test 2
SECTION A

Question 1 (c)
Question 2 (d)
Question 3 (e)
Question 4 (c)
Question 5 (d)
Question 6 (b)
Question 7 (d)
Question 8 (b)
SECTION B

Question 1

1.1 The moment $\overline{MO}$ is given by

$$\overline{MO} = \overline{DE} \times (150 \hat{i} - 0.6 \hat{j}) + \overline{OA} \times (30 \hat{j} N) - 24 \hat{j} Nm$$

$$= (0.2 \hat{k} m) \times (120 \hat{i} - 90 \hat{j}) N + (0.2 \hat{k} m) \times (30 \hat{j} N) - 24 \hat{j} Nm$$

$$= (24 \hat{j} + 18 \hat{i} - 6 \hat{i}) Nm - 24 \hat{j} Nm = 12 \hat{i} Nm$$

1.2 The equivalent system has resultant force $\overline{RO} = 30 \hat{j} N$ and resultant moment $M_O = 12 \hat{i} Nm$ at $O$ and $\overline{RO} \perp \overline{MO}$ since $\overline{RO} \cdot \overline{MO} = 0$. Hence the system consisting of $\overline{RO}$ and $\overline{MO}$ can be replaced by a single resultant force $\overline{RP} = \overline{RO}$, where $\overline{RP}$ acts at $P = (x, 0, z)$ and $M_O = \overline{OP} \times \overline{RO}$:

$$\begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
x & 0 & z \\
0 & 30 & 0 \\
\end{vmatrix} N = 12 \hat{i} Nm$$

$$\iff (-30z \hat{i} + 30x \hat{k}) N = 12 \hat{i} Nm$$

Hence $P = (0, 0, -0.4 m)$.

1.3 The equivalent system to the give force system is shown below.

The force $R_P = 30 \hat{j} N$ has a line of action parallel to the $y-$axis passing through point $P = (0, 0, -0.4 m)$. The line of action intersects the prism nowhere. Therefore, even if we use the fact that a force vector is a sliding vector, sliding $\overline{RP}$ along its line of action will never result in $\overline{RP}$ acting on a point on (or inside) the prism.
Question 2

FBD of A:

FBD of B:

Equilibrium equations:

\[ T_{AB} \cos \theta = \frac{2N_A}{\sqrt{13}} \]  
\[ T_{AB} \sin \theta = 100 N - \frac{3N_A}{\sqrt{13}} \]  
\[ T_{AB} \cos \theta = N_B \cos 50^\circ \]  
\[ T_{AB} \sin \theta = N_B \sin 50^\circ - 60 N \]

Solve for the unknowns from the 4 × 4 system of equations:

From Equations (1) and (3) it follows that
\[ \frac{2N_A}{\sqrt{13}} = N_B \cos 50^\circ \] and \[ 100 N - \frac{3N_A}{\sqrt{13}} = N_B \sin 50^\circ - 60 N. \]

Therefore \( N_A = 107.2 N \) and \( N_B = 92.47 N. \)

Substitute \( N_A = 107.2 N \) into Equations (1) and (2) to find
\( T_{AB} \cos \theta = 59.44 N \) and \( T_{AB} \sin \theta = 10.80 N. \)

Then \( \tan \theta = 0.1817 \) and hence \( \theta = 10.30^\circ. \)

From \( T_{AB} \cos \theta = 59.44 N \) it then follows that \( T_{AB} = \frac{59.44 N}{\cos 10.30^\circ} = 60.41 N. \)
Question 3

3.1 Free-body diagram

CD is a two-force member; hence no \( x \)– and \( y \)– force components.

No couples develop at \( E \); a proper alignment is in place.

\[ mg = 686.7 \text{ N} \]

3.2 Calculate support reactions.

Equilibrium equations:

\[
\begin{align*}
\sum F_x &= 0 \implies A_x + E_x = 0 \quad (1) \\
\sum F_y &= 0 \implies A_y = 0 \quad (2) \\
\sum F_z &= 0 \implies A_z + C_z + E_z = 686.7 \text{ N} \quad (3)
\end{align*}
\]

Take moments about \( A \)

\[
\bar{\mathbf{M}} = \mathbf{AC} \times (C_z \mathbf{k}) + \mathbf{AE} \times (E_z \mathbf{i} + E_z \mathbf{k}) + \mathbf{AG} \times (-686.7 \text{ N} \mathbf{k})
\]

\[
= (-0.2\mathbf{i} + 0.6\mathbf{j}) m \times (C_z \mathbf{k}) + 1.1\mathbf{j} m \times (E_z \mathbf{i} + E_z \mathbf{k}) + (0.3\mathbf{i} + 1.3\mathbf{j}) m \times (-686.7 \text{ N} \mathbf{k})
\]

\[
= \left[(0.6C_z + 1.1E_z - 892.7 \text{ N})\mathbf{i} + (0.2C_z + 206.0 \text{ N})\mathbf{j} + (-1.1E_x)\mathbf{k}\right] m
\]

Therefore

\[
\begin{align*}
\sum M_x &= 0 \implies 0.6C_z + 1.1E_z - 892.7 \text{ N} = 0 \quad (4) \\
\sum M_y &= 0 \implies 0.2C_z + 206.0 \text{ N} = 0 \quad (5) \\
\sum M_z &= 0 \implies -1.1E_x = 0 \quad (6)
\end{align*}
\]

Solve the \( 6 \times 6 \) system of equations:

From Equation (6) it follows that \( E_x = 0 \).

Substitute in Equation (1) to find \( A_x = 0 \).

From Equation (5) it follows that \( C_z = -1030 \text{ N} \).

Substitute in Equation (4) to find \( E_z = 1373 \text{ N} \).

Lastly, from Equation (3) it follows that \( A_z = 343.7 \text{ N} \).

Interpretation of the solution:
Question 4

Consider the free-body diagram of the simple truss below.

4.1 For equilibrium, check if a set of equilibrium equations hold:

\[ \sum F_x = A_x - D_x = 8 \text{kN} - 8 \text{kN} = 0 \]
\[ \sum F_y = A_y + G_y - H_y = 4.5 \text{kN} + 3 \text{kN} - 7.5 \text{kN} = 0 \]
\[ \sum M_{\text{point A}} = -(8 \text{m})H_y + (12 \text{m})G_y + (3 \text{m})D_x \\
= -(8 \text{m})(7.5 \text{kN}) + (12 \text{m})(3 \text{kN}) + (3 \text{m})(8 \text{kN}) \\
= -60 \text{kNm} + 36 \text{kNm} + 24 \text{kNm} \\
= 0 \]

Therefore the truss is in equilibrium, since the set of equations

\[ \sum F_x = 0, \quad \sum F_y = 0 \quad \sum M_{\text{point A}} = 0 \]

are satisfied.

4.2 The following members are zero-force members:

\( BK, \ DE, \ EG \) and \( GH \).

4.3 FBD node \( G \)

\[ F_{DG} = 3 \text{kN} \text{ (Compression)} \]

\[ 3 \text{kN} \]
FBD node C

\[ F_{CD} = 9.277 \text{ kN (Compression)} \]
\[ F_{CH} = 2 \left( \frac{0.277 \text{ kN}}{\sqrt{17}} \right) = 4.500 \text{ kN (Tension)} \]

FBD node K

\[ F_{KH} = 2 \text{ kN (Compression)} \]

FBD node H

\[ 0.8F_{BH} = 0.8F_{DH} + 2 \text{ kN} \]
\[ 0.6F_{BH} + 0.6F_{DH} = 3 \text{ kN} \]
\[ F_{BH} = 3.75 \text{ kN (Tension)} \]
\[ F_{DH} = 1.25 \text{ kN (Tension)} \]

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<th>Member</th>
<th>Magnitude (kN)</th>
<th>Tension</th>
<th>Compression</th>
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<td>Grootte (kN)</td>
<td>Trekkrag</td>
<td>Drukkrag</td>
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